Dear student following is a Moderate level [○●○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,−1). ((Questions may have more than one option correct).

Q.1 The tangent and normal to the curve $y = 2\sin x + \sin 2x$ are drawn at $P\left(x = \frac{\pi}{3}\right)$, then area of the quadrilateral formed by the tangent, the normal at P and the coordinate axis is-

(A) $\frac{\pi}{3}$ (B) 3π
(C) $\frac{\pi\sqrt{3}}{2}$ (D) None of these

Q.2 If $ax^2 + bx + c = 0$, a, b, c $\in$ R. Then the condition that this equation would have at least one root in (0, 1)
(A) $2a - 3b + 6c = 0$ (B) $2a + 3b + 6c = 0$
(C) $a + 3b + 6c = 0$ (D) $2a + 3b - 6c = 0$

Q.3 If $f(x) = x^{\alpha} \log x$ and $f(0) = 0$, then the value of 'α' for which Rolle's theorem can be applied in [0, 1] is-
(A) −2 (B) −1 (C) 0 (D) 1/2

Q.4 Total number of parallel tangents of $f_1(x) = x^2 - x + 1$ and $f_2(x) = x^3 - x^2 - 2x + 1$ are-
(A) 2 (B) 3
(C) Infinite (D) 0

Q.5 Let $f(x) = \int_0^x e^t (t-1)(t-2)\, dt$. Then f decreases in the interval-
(A) $(-\infty, -2)$ (B) $(-2, -1)$
(C) $(1, 2)$ (D) $(2, \infty)$

Q.6 If a < 0, and $f(x) = e^{ax} + e^{-ax}$ is monotonically decreasing. The interval to which x belongs-
(A) $x < 0$ (B) $x > 0$
(C) $x \le 0$ (D) $x \ge 0$

Q.7 Let $f(x) = 2x^3 - 9x^2 + 12x + 6$. Then the global maxima and global minima of f(x) in (1, 3) are-
(A) No, at x = 2 (B) At x = 2, no
(C) at x = 2, x = 1 (D) None of these

Q.8 If $0 < \alpha < \beta < \frac{\pi}{2}$, then $\alpha - \sin \alpha$
(A) $< \sin \beta - \beta$ (B) $< \beta - \sin \beta$
(C) $> \beta - \sin \beta$ (D) None of these

Q.9 The values of parameter 'a' for which the point of minimum of the function $f(x) = 1 + a^2x - x^3$ satisfies the inequality $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$ are-
(A) $\left(2\sqrt{3}, 3\sqrt{3}\right)$ (B) $\left(-3\sqrt{3}, -2\sqrt{3}\right)$
(C) $\left(-2\sqrt{3}, 3\sqrt{3}\right)$ (D) $\left(-3\sqrt{2}, 2\sqrt{3}\right)$

Q.10 Let $f'(\sin x) < 0$ and $f''(\sin x) > 0$, $\forall\, x \in \left(0, \frac{\pi}{2}\right)$ and $g(x) = f(\sin x) + f(\cos x)$, then g(x) is decreasing in-
(A) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (B) $\left(0, \frac{\pi}{4}\right)$
(C) $\left(0, \frac{\pi}{2}\right)$ (D) $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

MATHEMATICS IIT JEE (AUGUST 1st WEEK CLASS TEST 3) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	5	○	○	○	○	9	○	○	○	○
2	○	○	○	○	6	○	○	○	○	10	○	○	○	○
3	○	○	○	○	7	○	○	○	○					
4	○	○	○	○	8	○	○	○	○					

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	B	D	C	C	A	A	B	A,B	B

SOLUTIONS

Sol.1 (C)

Here, $\frac{dy}{dx} = 0$ at $\left(x = \frac{\pi}{3}, y = \frac{3\sqrt{3}}{2}\right)$

$\Rightarrow$ Tangent at $x = \frac{\pi}{3}$ is parallel to x-axis

$\Rightarrow$ Equation of tangent is, $y = \frac{3\sqrt{3}}{2}$

Also equation of normal is, $x = \frac{\pi}{3}$

$\therefore$ Area of quadrilateral

$$= \frac{\pi}{3} \cdot \frac{3\sqrt{3}}{2} = \frac{\pi\sqrt{3}}{2} \text{ sq. unit}$$

Sol.2 (B)

Let $f'(x) = ax^2 + bx + c$

Integrating both sides

$\Rightarrow$ $f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$

$\Rightarrow$ $f(0) = d$ and $f(1) = \frac{a}{3} + \frac{b}{2} + c + d$

Since Rolle's theorem is applicable

$\Rightarrow$ $f(0) = f(1)$

$\Rightarrow$ $d = \frac{a}{3} + \frac{b}{2} + c + d$

$\Rightarrow$ $2a + 3b + 6c = 0$ is the required condition

Sol.3 (D)

Clearly f(x) is continuous and differentiable on (0, 1) for $\alpha > 0$. Also, $f(0) = 0 = f(1)$

For f(x) to be continuous at x = 0, we must have

$$\lim_{x\to 0^+} f(x) = f(0), \text{ i.e., } \lim_{x\to 0^+} x^{\alpha} \log x = 0$$

Now, $\lim_{x\to 0^+} x^{\alpha} \log x = \lim_{x\to 0^+} \frac{\log x}{\frac{1}{x^{\alpha}}}$

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{-\alpha}{x^{\alpha+1}}} = 0 \; \{\text{as, } \alpha > 0\}$$

So, f(x) is continuous at x = 0 for $\alpha > 0$.

$\therefore$ By Rolle's theorem can be applied on f(x) in [0, 1] for all $\alpha > 0$.

Sol.4 (C)

Here, $f_1(x) = x^2 - x + 1$

and $f_2(x) = x^3 - x^2 - 2x + 1$

$\Rightarrow$ $f'_1(x_1) = 2x_1 - 1$

and $f'_2(x_2) = 3x_2^2 - 2x_2 - 2$

Let tangents drawn to the curves $y = f_1(x)$ and $y = f_2(x)$ at $(x_1, f_1(x_1))$ and $(x_2, f_2(x_2))$ are parallel,

$\Rightarrow$ $2x_1 - 1 = 3x_2^2 - 2x_2 - 2$

or $2x_1 - (3x_2^2 - 2x_2 - 1)$

Which is possible for infinite numbers of ordered pairs;

$\therefore$ Infinite solutions.

Sol.5 (C)

Here $f(x) = \int_0^x e^t (t - 1)(t - 2)\, dt$

$f'(x) = e^x (x - 1)(x - 2)$, {as e^x is always +ve} (using Leibnitz rule)

Using number line rule for f'(x) we get,

$f'(x) \le 0$ when $1 < x < 2$

$\therefore$ f decreases when $1 \le x \le 2$

Sol.6 (A)

Given a < 0, and .... (i)

$f(x) = e^{ax} + e^{-ax}$ is decreasing

$\Rightarrow$ $f'(x) < 0 \Rightarrow ae^{ax} - ae^{-ax} < 0$

$\Rightarrow$ $a\left(\frac{e^{2ax} - 1}{e^{ax}}\right) < 0$ .... (ii)

as from (i) a < 0

$\Rightarrow$ $(e^{2ax} - 1) > 0$ $\Rightarrow$ $e^{2ax} > 1$

$\Rightarrow$ $2ax > 0$ $\Rightarrow$ $ax > 0$

$\Rightarrow$ $x < 0$ {as a < 0}

Sol.7 (A)

$f(x) = 2x^3 - 9x^2 + 12x + 6$

$f'(x) = 6x^2 - 18x + 12$

$f'(x) = 6(x - 1)(x - 2) = 0$

$\Rightarrow$ $x = 1, 2$

$\therefore$ $f(1) = 11$ and $f(2) = 10$

Now let us consider the open interval (1, 3)
clearly x = 2 is the only point in (1, 3)
and f(2) = 10

Now $\lim_{x\to 1^+} f(x) = 11$ and $\lim_{x\to 3^-} f(x) = 15$

Thus x = 2 is the point of global minima in (1, 3) and global maxima does not exist in (1, 3).

Sol.8 (B)

Let $f(x) = x - \sin x$

$f'(x) = 1 - \cos x$

In interval $\left(0, \frac{\pi}{2}\right)$ $f'(x) = 1 - \cos x > 0$

Hence f(x) is an increasing function in the interval $\left[0, \frac{\pi}{2}\right]$

Now $0 < \alpha < \beta < \frac{\pi}{2}$

$\therefore$ $\alpha, \beta \in \left[0, \frac{\pi}{2}\right]$

$\therefore$ $\alpha < \beta \Rightarrow f(\alpha) < f(\beta)$

$\Rightarrow$ $\alpha - \sin\alpha < \beta - \sin\beta$

Sol.9 (A, B)

Given that $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$

$\Rightarrow$ $x \in (-3, -2)$

We have to find the extrema of the function
$f(x) = 1 + a^2x - x^3$

For maximum or minimum, f'(x) = 0

$\Rightarrow$ $a^2 - 3x^2 = 0$ or $x = \pm\frac{a}{\sqrt{3}}$ and $f''(x) = -6x$ is +ve when x is negative.

If a is positive then point of minima is $-\frac{a}{\sqrt{3}}$

i.e. $-3 < -\frac{a}{\sqrt{3}} < -2$ or $2\sqrt{3} < a < 3\sqrt{3}$

And if a is negative then point of minima is $\frac{a}{\sqrt{3}}$

i.e. $-3 < \frac{a}{\sqrt{3}} < -2$ or $-3\sqrt{3} < a < -2\sqrt{3}$

Then, $a \in \left(-3\sqrt{3}, -2\sqrt{3}\right) \cup \left(2\sqrt{3}, 3\sqrt{3}\right)$

Sol.10 (B)

$g'(x) = f'(\sin x) \cdot \cos x - f'(\cos x) \cdot \sin x$

$\Rightarrow$ $g''(x) = -f'(\sin x)\sin x + \cos^2 x \cdot f''(\sin x) + f''(\cos x) \cdot \sin^2 x - f'(\cos x) \cdot \cos x > 0 \ \forall \ x \in \left(0, \frac{\pi}{2}\right)$

$\Rightarrow$ g'(x) is increasing in $\left(0, \frac{\pi}{2}\right)$.

Also $g'\left(\frac{\pi}{4}\right) = 0$

$\Rightarrow$ $g'(x) > 0 \ \forall \ x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

and $g'(x) < 0 \ \forall \ x \in \left(0, \frac{\pi}{4}\right)$.

Thus g(x) is decreasing in $\left(0, \frac{\pi}{4}\right)$.