Dear student following is a Moderate level [○ ● ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (All questions have only one option correct).

Q.1 A point P is given on the circumference of a circle of radius r. The chord QR is parallel to the tangent line at P. The maximum area of the triangle PQR is

(A) $\frac{3\sqrt{2}}{4}$ r^2 (B) $\frac{3\sqrt{3}}{4}$ r^2

(C) $\frac{3}{8}$ r (D) None

Q.2 A tangent is drawn at the point $(3\sqrt{3}\cos\theta, \sin\theta)$ $0 < \theta < \frac{\pi}{2}$ of an ellipse $\frac{x^2}{27} + \frac{y^2}{1} = 1$ the least value of the sum of the intercepts the co-ordinate axes by this tangent is attained at θ =

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{8}$ (D) $\frac{\pi}{4}$

Q.3 If $y = a \log|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

(A) a = 2, b = –1 (B) a = 2, b = –1/2

(C) a = –2, b = 1/2 (D) None of these

Q.4 In a sub-marine telegraph-cable the speed of signalling varies as $x^2 \log(1/x)$ where x is the ratio of the radius of the case to that of covering. The greatest speed is attained when this ratio is

(A) 1 : e (B) 1 : $\sqrt{e}$

(C) 1 : $e\sqrt{e}$ (D) None

Q.5 The value of n for which the length of the subnormal of the curve $xy^n = a^{n+1}$ is constant

(A) 1 (B) –1

(C) 2 (D) –2

Q.6 The triangle formed by the tangent to the curve $f(x) = x^2 + bx - b$ at the point (1, 1) and the co-ordinate axes, lies in the first quadrant. If its area is 2, then the value of b is

(A) –1 (B) 3

(C) –3 (D) 1

Q.7 Tangents to the folium of descartes $x^3 + y^3 = 3axy$ at the point where it meets the parabola $y^2 = ax$ are parallel to

(A) x-axis (B) y-axis

(C) y = x (D) None

Q.8 The straight line x + y = a will be a tangent to the ellipse $x^2/9 + y^2/16 = 1$ if a =

(A) 8 (B) ± 5

(C) ± 10 (D) ± 6

Q.9 If tangent at any point on the curve $e^y = 1 + x^2$ makes an angle θ with +ive direction of x-axis, then

(A) $|\tan\theta| > 1$ (B) $|\tan\theta| < 1$

(C) $\tan\theta > 1$ (D) $|\tan\theta| \leq 1$

Q.10 A curve has C has the property that if the tangent drawn at any point P on C meets the co-ordinate axes at A and B, then P is the mid-point of AB. The curve passes through the point (1, 1). The equation of the curve is

(A) xy = 1 (B) $y^2 = 2x - 1$

(C) $x^2 = 2y - 1$ (D) None

MATHEMATICS IIT JEE (AUGUST 1st WEEK CLASS TEST 4) (DERIVATE & IT'S APP.) ANSWER KEY

Name : ..... **Roll No. :** .....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	5	○	○	○	○	9	○	○	○	○
2	○	○	○	○	6	○	○	○	○	10	○	○	○	○
3	○	○	○	○	7	○	○	○	○					
4	○	○	○	○	8	○	○	○	○					

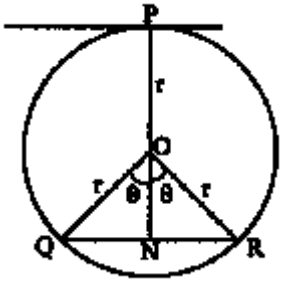
ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A	B	B	D	C	B	B	D	A

SOLUTIONS

Sol.1 (B)

$A = \frac{1}{2}$. QR. PN $= \frac{1}{2}$ 2r sinθ . (r + r cosθ)



$A = r^2\left(\sin\theta + \frac{1}{2}\sin 2\theta\right)$ and it will be maximum

when $\cos\theta + \cos 2\theta = 0$

or $\cos 2\theta = -\cos\theta = \cos(\pi - \theta)$ or when

$\theta = \pi/3$. In this case the triangle will be equilateral and its area will be

$r^2\left(\sin\frac{\pi}{3} + \frac{1}{2}\sin\frac{2\pi}{3}\right) = \frac{3\sqrt{3}}{4}\ r^2$

Sol.2 (A)

Tangent is $\frac{x\cos\theta}{3\sqrt{3}} + \frac{y\sin\theta}{1} = 1$

y = sum of intercepts = $3\sqrt{3}\sec\theta + \text{cosec}\,\theta$

For min., $\frac{dy}{d\theta} = 0$

$\therefore \quad 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\,\theta\,.\cot\theta = 0$

$\therefore \quad \tan^3\theta\ \frac{1}{3\sqrt{3}} \qquad \therefore \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = \frac{\pi}{6}$

Sol.3 (B)

$\frac{dy}{dx} = 0$ or $\frac{a}{x} + 2bx + 1 = 0$ at x –1, 2

$\therefore \quad a + 2b = 1 \qquad$ and $\quad a + 8b = -2$

on solving above equations

we get $a = 2,\ b = -\frac{1}{2}$

Sol.4 (B)

$S = k.\ x^2 \log\frac{1}{x} = -kx^2\log x$

$\frac{ds}{dx} = -k\left[x^2.\frac{1}{x} + 2x\log x\right] = 0$

or $\quad -kx(1 + 2\log x) = 0$

$\therefore \quad x = 0$ or $\log x = -\frac{1}{2}$

$\therefore \quad x = e^{-1/2} = \frac{1}{\sqrt{e}}$

$\frac{d^2s}{dx^2} = -k\left(1 + 2\log x + x.\frac{2}{x}\right)$

$= -k(3 + 2\log x)$

$= -k(3 - 1) = -2k = -\text{ive}$

and hence S is maximum when $x = \frac{1}{\sqrt{e}}$.

Sol.5 (D)

$\log x + n\log y = (n + 1)\log a$

$\therefore \ \frac{1}{x} + \frac{n}{y}.y' = 0 \qquad \therefore\ y' = -\frac{y}{nx}$

S.N. = $yy' = -\frac{y^2}{nx} = \frac{y^2y^n}{na^{n+1}} = -\frac{y^{n+2}}{na^{n+1}}$

It will be constant if n + 2 = 0

$\therefore \quad n = -2$

Sol.6 (C)

$\frac{dy}{dx} = 2x + b = 2 + b$ at (1, 1)

Equation of tangent is y – 1 = (2 + b)(x – 1)

Its intercepts A and B on the axes are obtained by putting y = 0 and then x = 0

$\therefore \quad A = 1 - \frac{1}{2+b} = \frac{b+1}{b+2}$

$B = 1 - (2 + b) = -(b + 1)$

$\Delta = \frac{1}{2}AB = 2 \quad \therefore AB = 4$

$-(b+1)(b+1) = 4(b+2)$

or $b^2 + 6b + 9 = 0$

or $(b+3)^3 = 0 \quad \therefore b = -3$

Sol.7 (B)

$x^3 + y^3 - 3axy = 0$

or $\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{(x^2 - ay)}{(y^2 - ax)}$

At the points where the curve meets $y^2 = ax$, the value of $\frac{dy}{dx} = \frac{x^2 - ay}{0} = \infty$ and hence tangent is perpendicular to x-axis or parallel to y-axis.

Sol.8 (B)

If the given line is a tangent to ellipse it will cut it in two coincident points. Eliminating y, we get $25x^2 - 18ax + 9(a^2 - 16) = 0$

$\therefore \quad B^2 - 4AC = 0$

or $18 \times 18a^2 - 4 \times 25 \times 9(a^2 - 16) = 0$

$\therefore \quad 16a^2 = 400 \quad$ or $\quad a = \pm 5$

Sol.9 (D)

$y = \log(1 + x^2)$

$m = \tan\theta = \frac{dy}{dx} = \frac{2x}{1 + x^2}$

$|\tan\theta| = \left|\frac{2x}{1 + x^2}\right| = \frac{2|x|}{1+|x|^2} \le 1$

$1 + |x|^2 \ge 2|x|$ by A.M. $\ge$ G.M.

$\therefore \frac{2|x|}{1+|x|^2} \le 1.$

Sol.10 (A)

$Y - y = \frac{dy}{dx}(X - x)$ is equation of tangent at P(x, y). Putting Y = 0 , then X = 0.

$A = \left(x - y / \frac{dy}{dx}, 0\right)$; $B = \left(0, y - x\frac{dy}{dx}\right)$

P is mid point of AB

$\therefore \quad 2x = x - y/\frac{dy}{dx}$ and $2y = y - x\frac{dy}{dx}$

$\therefore \quad -\frac{y}{x} = \frac{dy}{dx}$ from either

or $xdy + y\,dx = 0 \quad$ or $\quad dx/x + dy/y = 0$

Intergrating $xy = \lambda$. It passes through (1, 1).

$\therefore \quad \lambda = 1$

$\therefore \quad xy = 1$ is the required equation of the curve.