Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 15 Marks in 15 Minutes would be a satisfactory performance. Questions 1-9(+3,–1). (All questions have only one option correct).

Q.1 Let f(x) be a polynomial function of second degree. If f(1) = f(–1) and a_1, a_2, a_3 are in A.P. Then $f'(a_1)$, $f'(a_2)$, $f'(a_3)$ are in

(A) A.P. (B) G.P. (C) H.P. (D) None

Q.2 If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then $\frac{dy}{dx}$ =

(A) $\frac{2(1+x+x^2)}{(x^2+1)}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(B) $\frac{2(1-x+x^2)}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(C) $\frac{2(1+x-x^2)}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$

(D) $\frac{2(1+x-x^2)}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)$

Q.3 Let f be twice differentiable such that f''(x) = –f(x) and f'(x) = g(x). If $h(x) = \{f(x)\}^2 + \{g(x)\}^2$, where h(5) = 11, then h(10) =

(A) 0 (B) 11 (C) 1/11 (D) C

Q.4 If y = f(x), then $\frac{d^2x}{dy^2}$ in terms of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)}$ (B) $\frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$ (C) $\frac{\left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}}$ (D) $\frac{-\left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}}$

Q.5 If g is the inverse function of f and $f'(x) = \frac{1}{1+x^n}$ then g'(x) =

(A) $\frac{1}{1+[g(x)]^n}$ (B) $1 + [g(x)]^n$

(C) $1 + \frac{1}{[g(x)]^n}$ (D) $\frac{[g(x)]^n}{1+[g(x)]^n}$

Q.6 If $y = \tan\left(\frac{1}{2}\cos^{-1}\frac{1-u^2}{1+u^2} + \frac{1}{2}\sin^{-1}\frac{2u}{1+u^2}\right)$ and $x = \frac{2u}{1-u^2}$, then $\frac{dy}{dx}$ =

(A) –1 (B) 0 (C) 1 (D) None

Q.7 If $y = \cot^{-1}(\cos 2x)^{1/2}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ will be

(A) $\left(\frac{2}{3}\right)^{1/2}$ (B) $\left(\frac{1}{3}\right)^{1/2}$ (C) $(3)^{1/2}$ (D) $(6)^{1/2}$

Q.8 Let f(x) and g(x) be two functions having finite non-zero 3rd order derivatives f'''(x) and g'''(x) for all, $x \in R$. If f(x)g(x) = 1 for all $x \in R$, then $\frac{f'''}{f'} - \frac{g'''}{g'}$ is equal to

(A) $3\left(\frac{f''}{g} - \frac{g''}{f}\right)$ (B) $3\left(\frac{f''}{f} - \frac{g''}{g}\right)$

(C) $3\left(\frac{g''}{g} - \frac{f''}{g}\right)$ (D) $3\left(\frac{f''}{f} - \frac{g''}{f}\right)$

Q.9 $\frac{d}{dx}\left[\tan^{-1}\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ =

(A) $\frac{-x}{\sqrt{1-x^4}}$ (B) $\frac{x}{\sqrt{1-x^4}}$

(C) $\frac{-x}{2\sqrt{1-x^4}}$ (D) $\frac{1}{2\sqrt{1-x^4}}$

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 3) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... **Roll No. :** ....

	A	B	C	D		A	B	C	D		A	B	C	D
1	○	○	○	○	4	○	○	○	○	7	○	○	○	○
2	○	○	○	○	5	○	○	○	○	8	○	○	○	○
3	○	○	○	○	6	○	○	○	○	9	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9
Ans.	A	C	B	B	B	C	A	B	A

SOLUTIONS

Sol.1 (A)

Let $f(x) = \lambda x^2 + \mu x + \nu$

$\Rightarrow$ $f'(x) = 2\lambda x + \mu$

Also $f(1) = f(-1)$

$\Rightarrow$ $\lambda + \mu + \nu = \lambda - \mu + \nu$

$\Rightarrow$ $\mu = 0$

$\therefore$ $f'(a_1) = 2\lambda a_1$, $f'(a_2) = 2\lambda a_2$, $f'(a_3) = 2\lambda a_3$

as a_1, a_2 and a_3 are in A.P.

$\Rightarrow$ $f'(a_1), f'(a_2), f'(a_3)$ are also in A.P.

Sol.2 (C)

$$\frac{dy}{dx} = \frac{d}{dx} f\left(\frac{2x-1}{x^2+1}\right)$$

$$= f'\left(\frac{2x-1}{x^2+1}\right)\left\{\frac{(x^2+1)2-(2x-1)2x}{(x^2+1)^2}\right\}$$

$$= \sin\left(\frac{2x-1}{x^2+1}\right)^2 \frac{2(x^2+1-2x^2+x)}{(x^2+1)^2}$$

$$= \frac{2(1+x-x^2)}{(x^2+1)^2}\sin\left(\frac{2x-1}{x^2+1}\right)^2$$

Sol.3 (B)

Given, $h(x) = \{f(x)\}^2 + \{g(x)\}^2$

On differentiating both sides w.r.t. x, we get

$h'(x) = 2f(x).f'(x) + 2g(x).g'(x)$ ....(i)

Now $f'(x) = g(x)$.

then, $f''(x) = g'(x)$

$\Rightarrow$ $-f(x) = g'(x)$ ....(ii)

$\{\because f''(x) = -f(x), \text{given}\}$

from (i) and (ii), we get,

$h'(x) = 2f(x).g(x) + 2g(x).\{-f(x)\}$

{using $f'(x) = g(x)$ and $g'(x) = -f(x)$}

$\therefore$ $h'(x) = 0$

So, h(x) must be constant $\left[\text{as } \frac{d}{dx}\text{constant} = 0\right]$

but $h(5) = 11$. So, $h(x) = 11$

Hence, $h(10) = 11$.

Sol.4 (B)

As $$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{1}{dy/dx}\right)$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{d}{dx}\left(\frac{1}{dy/dx}\right)\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = -\frac{1}{\left(\frac{dy}{dx}\right)^2}\frac{d}{dx}\left(\frac{dy}{dx}\right).\frac{dx}{dy}$$

$$\Rightarrow \frac{d^2x}{dy^2} = \frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

Sol.5 (B)

As g is inverse function of f(x).

So we can take $g(x) = f^{-1}(x)$

$\Rightarrow$ $f(g(x)) = x$

$\Rightarrow$ $f'[g(x)]\, g'(x) = 1$

$$\Rightarrow g'(x) = \frac{1}{f'[g(x)]} = \frac{1}{\frac{1}{1+[g(x)]^n}}$$

$\Rightarrow$ $g'(x) = 1 + [g(x)]^n$

Sol.6 (C)

$$y = \tan\left(\frac{1}{2}(2\tan^{-1}u) + \frac{1}{2}(2\tan^{-1}u)\right)$$

$= \tan(\tan^{-1}u + \tan^{-1}u)$

$= \tan(2\tan^{-1}u)$

$$= \tan\left(\tan^{-1}\frac{2u}{1-u^2}\right) = \frac{2u}{1-u^2} = x$$

$$\therefore \frac{dy}{dx} = 1$$

Sol.7 (A)

$$\frac{dy}{dx} = -\frac{-1}{1+\cos 2x} \times \frac{1}{2\sqrt{\cos 2x}} \times -2\sin 2x$$

$$= \frac{\tan x}{\sqrt{\cos 2x}}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\pi/6} = \frac{1/\sqrt{3}}{\sqrt{1/2}} = \sqrt{\frac{2}{3}}.$$

Sol.8 (B)

We have f(x)g(x) = 1

Differentiating with respect to x, we get

$f'g + fg' = 0$ .....(i)

Differentiating (i), w.r.t. x, we get

$f''g + 2f'(g)' + fg'' = 0$ .....(ii)

Differentiating (ii) w.r.t. x, we get

$f'''g + g'''f + 3f''g' + 3g''f' = 0$

$$\Rightarrow \frac{f'''}{f'}(f'g) + \frac{g'''}{g'}(fg') + \frac{3f''}{f}(fg') + \frac{3g''}{g}(gf')$$

$$\Rightarrow -\left(\frac{f'''}{f'} + \frac{3g''}{g}\right)(fg') = -\left(\frac{g'''}{g'} + \frac{3f''}{f}\right)f(g'),$$

[using (i)]

$$\Rightarrow \frac{f'''}{f'} - \frac{g'''}{g'} = 3\left(\frac{f''}{f} - \frac{g''}{g}\right).$$

Sol.9 (A)

On putting $x^2 = \cos 2\theta$, we get

$$A = \frac{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}$$

$$A = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$

$$A = \frac{1+\tan\theta}{1-\tan\theta} = \tan\left(\theta + \frac{\pi}{4}\right)$$

Now $y = \tan^{-1}\tan\left(\theta + \frac{\pi}{4}\right)$

$$= \theta + \frac{\pi}{4} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$$

$$\therefore \quad \frac{dA}{dx} = \frac{-x}{\sqrt{1-x^4}}$$