Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-10(+3,–1). (Questions may have more than one option correct).

Q.1 The equation of normal to the curve $x + y = x^y$, where it cuts x-axis, is :

(A) $y = x$ (B) $y = x + 1$

(C) $y = x - 1$ (D) $x + y = 1$

Q.2 If c be a positive constant and $|f(y) - f(x)| \le c\,(y - x)^2$ for all real x and y, then :

(A) $f(x) = 0$ for all x (B) $f(x) = x$ for all x

(C) $f'(x) = 0$ for all x (D) $f'(x) = c$ for all x

Q.3 Let $f : R \to R$ be a function such that $f(x) = ax + 3 \sin x + 4 \cos x$. Then f(x) is invertible if-

(A) $a \in (-5, 5)$ (B) $a \in (-\infty, -5)$

(C) $a \in (5, \infty)$ (D) None of these

Q.4 If $f(x) = \cos\left\{\frac{\pi}{2}[x] - x^3\right\}$, $1 < x < 2$ and [x] = the greatest integer $\le$ x, then $f'\left(\sqrt[3]{\frac{\pi}{2}}\right)$ is equal to-

(A) 0 (B) $3\left(\frac{\pi}{2}\right)^{2/3}$

(C) $\left(\frac{\pi}{2}\right)^{2/3}$ (D) None of these

Q.5 Let f and g be functions from the interval $[0, \infty)$ to the interval $[0, \infty)$, f being an increasing and g being a decreasing function. If $f\{g(0)\} = 0$ then-

(A) $f\{g(x)\} \ge f\{g(0)\}$ (B) $g\{f(x)\} \le g\{f(0)\}$

(C) $f\{g(2)\} = 0$ (D) None of these

Q.6 Let $F(x) = f(x)\, g(x)\, h(x)$ for all x, where g(x) and h(x) are differentiable functions. At some point x_0, $F'(x_0) = 21F(x_0)$, $f'(x_0) = 4f(x_0)$, $g'(x_0) = -7g(x_0)$ and $h'(x_0) = kh(x_0)$. Then k is-

(A) 1 (B) 7 (C) 24 (D) 5

Q.7 If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation-

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$

(C) $2 \le M \le 3$ (D) $3 \le M \le 4$

Q.8 If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$

(A) f(x) is a strictly increasing function

(B) f(x) has a local maxima

(C) f(x) is a strictly decreasing function

(D) f(x) is bounded

Q.9 A wire of length $\ell = 6 \pm 0.06$ cm and radius $r = 0.5 \pm 0.005$ cm and mass $m = 0.3 \pm 0.003$ gm. Maximum percentage error in density is-

(A) 4 (B) 2 (C) 1 (D) 6.8

Q.10 A function y = f(x) has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is-

(A) $(x + 1)^2$ (B) $(x - 1)^3$

(C) $(x + 1)^3$ (D) $(X - 1)^2$.

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 4) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D
1	○	○	○	○
2	○	○	○	○
3	○	○	○	○
4	○	○	○	○
5	○	○	○	○
6	○	○	○	○
7	○	○	○	○
8	○	○	○	○
9	○	○	○	○
10	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	A	A	B,C	C	A	A	A	B

SOLUTIONS

Sol.1 (C)

The given curve $x + y = x^y$ .... (1)

It cuts the x-axis, when $y = 0 \Rightarrow x = 1$

$\therefore$ The required point is (1, 0)

Taking log of (1), we get

$\log(x + y) = y \log x$

Differentiating w.r.t. x,

$$\frac{1}{x+y}\left[1+\frac{dy}{dx}\right] = \frac{dy}{dx}\log x + \frac{y}{x}$$

When, x = 1, y = 0, we get

$$\frac{1}{1+0}\left[1+\frac{dy}{dx}\right] = \frac{dy}{dx} \times 0 + \frac{0}{1}$$

$$\therefore \quad \frac{dy}{dx} = -1.$$

So, the slope of normal = 1.

Equation of the normal is $y - 0 = 1(x - 1)$

$\Rightarrow \quad y - x + 1 = 0.$

Sol.2 (C)

Given, $|f(y) - f(x)| \le c(y - x)^2 \ \forall \ x \in R$

$$\Rightarrow \left|\frac{f(x+h)-f(x)}{h}\right| \le c\,|h| \ \forall \ x$$

(Put y = x + h)

$$\therefore \lim_{h\to 0}\left|\frac{f(x+h)-f(x)}{h}\right| \le \lim_{h\to 0} c\,|h|$$

$\Rightarrow \quad |f'(x)| \le 0 \ \forall \ x$

$\Rightarrow \quad f'(x) = 0 \ \forall \ x \quad [\because |x| \not< 0]$

Sol.3 (A)

$f(x) = ax + 3\sin x + 4\cos x$

$\Rightarrow \quad f'(x) = a + 3\cos x - 4\sin x$

Put $3 = r\cos\alpha$, $4 = -r\sin\alpha$

$$\Rightarrow \quad r = 5 \therefore 3 = 5\cos\alpha \Rightarrow \cos\alpha = \frac{3}{5}$$

Now, $f'(x) = a + r\cos(x + \alpha)$

$= a + 5\cos(x + \alpha)$

Since, $-1 \le \cos(x + \alpha) \le 1$

$\Rightarrow \quad a - 5 \le f'(x) \le a + 5$

$\therefore \quad f'(x) > 0$ if $a + 5 > 0$, i.e., $a > -5$,

and $\quad f'(x) < 0$ if $a - 5 < 0$, i.e., $a < 5$

Hence, f(x) is strictly monotonic if $a \in (-5, 5)$ and hence, it will be invertible.

Sol.4 (A)

$\because \quad 1 < x < 2 \Rightarrow [x] = 1$

$$\therefore \quad f(x) = \cos\left(\frac{\pi}{2} - x^3\right) = \sin x^3$$

$\Rightarrow \quad f'(x) = 3x^2\cos x^3$

$$f'\left(\sqrt[3]{\frac{\pi}{2}}\right) = 3\left(\frac{\pi}{2}\right)^{2/3}\cos\frac{\pi}{2} = 0$$

Sol.5 (B,C)

$f'(x) > 0$ if $x \ge 0$ and $g'(x) < 0$ if $x \ge 0$

Let $h(x) = f(g(x))$ then $h'(x)$

$= f'(g(x)).g'(x) < 0$ if $x \ge 0$

$\therefore \quad h(x)$ is decreasing function

$\therefore \quad h(x) \le h(0)$ if $x \ge 0$

$\therefore \quad f(g(x)) \le f(g(0)) = 0$

But codomain of each function is $[0, \infty)$

$\therefore \quad f(g(x)) = 0$ for all $x \ge 0$

$\therefore \quad f(g(x)) = 0$

Also $g(f(x) \le g(f(0))$ [as above]

Sol.6 (C)

$F(x) = f(x)\,g(x)\,h(x)$

$\Rightarrow \quad F'(x) = f'(x)\,g(x)\,h(x) + f(x)\,g'(x)\,h(x) + f(x)\,g(x)\,h'(x)$

$\Rightarrow \quad F'(x_0) = f'(x_0)\,g(x_0)\,h(x_0) + f(x_0)\,g'(x_0)\,h(x_0) + f(x_0)\,g(x_0)\,h'(x_0)$

$21F(x_0) = 4f(x_0)\,g(x_0)\,h(x_0) + f(x_0)\,(-7)\,g(x_0)\,h(x_0) + f(x_0)\,g(x_0)\,kh(x_0)$

$21f(x_0)\,g(x_0)\,h(x_0) = (4 - 7 + k)\,f(x_0)\,g(x_0)\,h(x_0)$

$\Rightarrow \quad 21 = 4 - 7 + k \quad \Rightarrow k = 24.$

Sol.7 (A)

As A.M. $\geq$ G.M. for positive real numbers, we get

$$\frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)\,(c+d)}$$

$\Rightarrow$ M $\leq$ 1 (putting values)

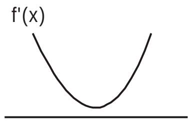
Also, (a + b) (c + d) > 0 [$\because$ a, b, c, d > 0]

$\therefore$ 0 $\leq$ M $\leq$ 1.

Sol.8 (A)

$f(x) = x^3 + bx^2 + cx + d \quad 0 < b^2 < c$

$f'(x) = 3x^2 + 2bx + c$



Discriminant = $4b^2 - 12c = 4(b^2 - 3c) < 0$

$\therefore$ f'(x > 0 $\forall$ x $\in$ R

$\Rightarrow$ f(x) is strictly increasing $\forall$ x $\in$ R

Sol.9 (A)

$$\rho = \frac{m}{\ell\pi r^2}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{2\Delta r}{r} + \frac{\Delta\ell}{\ell}$$

Putting the values $\Delta\ell$ = 0.06 cm

ℓ = 6cm; Δ r = 0.005 cm, r = 0.5 cm

m = 0.3 gm; Δ m = 0.003 gm

We get $\frac{\Delta\rho}{\rho} = \frac{4}{100}$

$\therefore \quad \frac{\Delta\rho}{\rho} \times 100 = 4\%$

Sol.10 (B)

$f''(x) = 6(x - 1)$. Integrating, we get

$f'(x) = 3x^2 - 6x + c$

Slope at (2, 1) = f'(2) = c = 3

[$\because$ slope of tangent at (2, 1) is 3]

$\therefore \quad f'(x) = 3x^2 - 6x + 3 = 3(x - 1)^2$

Integrating again, we get,

$f(x) = (x - 1)^3 + D$

The curve passes through (2, 1)

$\Rightarrow \quad 1 = (2 - 1)^3 + D \Rightarrow D = 0$

$\therefore \quad f(x) = (x - 1)^3$