Dear student following is a Moderate level [○ ○ ● ○ ○] test paper. Score of 18 Marks in 15 Minutes would be a satisfactory performance. Questions 1-8(+3,–1) & 9(+6, 0). (Questions may have more than one option correct).

Q.1 The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical is (Are)-

(A) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (B) $\left(\pm\sqrt{\frac{11}{3}}, 1\right)$

(C) (0, 0) (D) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

Q.2 The length of a longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing is-

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) π

Q.3 The equation $e^{x-1} + x - 2 = 0$ has-

(A) One real root (B) Two real roots

(C) Three real roots (D) Four real roots

Q.4 Suppose f'(x) exists for each x and $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then

(A) h is increasing whenever f is increasing

(B) h is increasing whenever f is decreasing

(C) h is decreasing whenever f is decreasing

(D) Nothing can be said in general.

Q.5 If $f(x) = \frac{a^2-1}{a^2+1} x^3 - 3x + 5$ is a decreasing function of x in R then the set of possible values of a (independent of x) is-

(A) $(1, \infty)$ (B) $(-\infty, -1)$

(C) [–1, 1] (D) None of these

Q.6 If f(x) be a differentiable function such that f(xy) = f(x) + f(y) for all x and y, then f(e) + f(1/e) =

(A) 1 (B) 0 (C) –1 (D) None

Q.7 Let $f(x) = \sqrt{x-1} + \sqrt{x+24-10\sqrt{x-1}}$; $1 < x < 26$ be real valued function. Then f'(x) for 1 < x < 26 is-

(A) 0 (B) $\frac{1}{\sqrt{x-1}}$

(C) $2\sqrt{x-1} - 5$ (D) None of these

Q.8 Area of the triangle formed by the normal to the curve $x = e^{\sin y}$ at (1, 0) with the coordinate axes is :

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

Q.9 Match the column

Let $f(x) = (2^x - 1)(2^x - 2)$ and

$g(x) = 2 \sin x + \cos 2x$

Column I	Column II
(I) f increases on	(A) (π, ∞)
(II) f decreases on	(B) $(-\infty, \pi)$
(III) g decreases on	(C) $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$
(IV) g increases on	(D) $\left(0, \frac{\pi}{6}\right)$

MATHEMATICS IIT JEE (JULY 2nd WEEK CLASS TEST 5) (DERIVATE & IT'S APP.) ANSWER KEY

Name : .... Roll No. : ....

	A	B	C	D		A	B	C	D	9	A	B	C	D
1	○	○	○	○	5	○	○	○	○	(I)	○	○	○	○
2	○	○	○	○	6	○	○	○	○	(II)	○	○	○	○
3	○	○	○	○	7	○	○	○	○	(III)	○	○	○	○
4	○	○	○	○	8	○	○	○	○	(IV)	○	○	○	○

ANSWER KEY

Que.	1	2	3	4	5	6	7	8
Ans.	D	A	A	A,C	C	B	A	B

9. (I-A, II-B, III-D, IV-C)

SOLUTIONS

Sol.1 (D)

Differentiating the given curve w.r.t. x, we get

$$3y^2\frac{dy}{dx} + 6x = 12\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{2x}{y^2-4}$$

At point where the tangent(s) is (are) vertical,

$\frac{dy}{dx}$ is not defined, i.e. at those points,

$$y^2 - 4 = 0 \Rightarrow y = \pm 2$$

When y = 2, $8 + 3x^2 = 12(2)$

$$\Rightarrow \quad 3x^2 = 16 \Rightarrow x = \pm\frac{4}{\sqrt{3}}$$

When y = – 2, $-8 + 3x^2 = -24$

$\Rightarrow \quad 3x^2 = -16$. This is not possible.

Thus, the required points are $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

Sol.2 (A)

Since $3\sin x - 4\sin^3 x = \sin 3x$ and sin 3x increases when 3x takes values from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ or x takes values from $-\frac{\pi}{6}$ to $\frac{\pi}{6}$. Thus, the length of the longest interval in which $3\sin x - 4\sin^3 x$ increases is $\frac{\pi}{6} - (-\frac{\pi}{6}) = \frac{\pi}{3}$.

Sol.3 (A)

Clearly, x = 1 satisfies the given equation. Assume that $f(x) = e^{x-1} + x - 2 = 0$ has a real root α other that x = 1. We may suppose that $\alpha > 1$ (the case $\alpha < 1$ is exactly similar). Applying Rolle's theorem on $[1, \alpha]$ (if $\alpha < 1$ apply the theorem on $[\alpha, 1]$), we get $\beta \in (1, \alpha)$ such that $f'(\beta) = 0$. But $f'(\beta) = e^{\beta-1} + 1$, so that $e^{\beta-1} = -1$, which is not possible. Hence there is no real root other than 1.

Sol.4 (A, C)

We have

$h'(x) = f'(x) - 2f'(x) f(x) + 3f'(x) f(x)^2$

$$= 3f'(x)\left[f(x)^2 - \frac{2}{3}f(x) + \frac{1}{3}\right]$$

$$= 3f'(x)\,[(f(x)\ \ 1/3)^2 + 2/9]$$

Thus, $h'(x) > 0$ if $f'(x) > 0$

and $h'(x) < 0$ if $f'(x) < 0$

Therefore, h increases (decreases) whenever f increases (decreases).

Sol.5 (C)

$$f'(x) = 3\left(\frac{a^2-1}{a^2+1}\right)x^2 - 3$$

$f'(x) < 0$ for all x if $a^2 - 1 \le 0$

$\Rightarrow \quad -1 \le a \le 1$

Sol.6 (B)

Put x = y = 1, we get f(1) = 0

Put $y = \frac{1}{x}$ we get $f(x) + f\left(\frac{1}{x}\right) = f(1) = 0$

$$\therefore \quad f(e) + f\left(\frac{1}{e}\right) = 0$$

Sol.7 (A)

$$f(x) = \sqrt{x-1} + \sqrt{25 + (x-1) - 10\sqrt{x-1}}$$

$$= \sqrt{x-1} + \sqrt{(5-\sqrt{x-1})^2}$$

$$= \sqrt{x+1} + \left|5 - \sqrt{x-1}\right| = 5$$

$$\left[\because \sqrt{x-1} < 5 \text{ for } 1 < x < 26\right]$$

$$\therefore \quad f'(x) = 0$$

Sol.8 (B)

$x = e^{\sin y} \Rightarrow \ln x = \sin y$

$\Rightarrow \frac{1}{x}\frac{dx}{dy} = \cos y$

$\therefore$ slope of normal at (1, 0)

$= -\frac{dx}{dy} = -1 \times \cos 0 = -1$

$\therefore$ Equation of normal is $y - 0$

$= -1(x - 1) \Rightarrow x + y = 1$

Which forms triangle of area $\frac{1}{2}$ with axes.

Sol.9 (I-A, II-B, III-D, IV-C)

$y'(x) = 2^x (2^{x+1} - 3) \log 2$.

Since $2^x > 0$ and $\log 2 > 0$ so $y'(x) > 0$ if $2x + 1 - 3 > 0$, i.e. $x + 1 > \log (3/2) = \log_2 3$. Thus $y'(x) > 0$ for $x > \log_2 3/2$.

Hence y increases on $(\log_2 3/2, \infty)$ and decreases on $(-\infty, \log_2 3/2) \supset (-\infty, -\pi)$

The period of g is 2π so it is enough to consider g on $[0, 2\pi]$.

$g'(x) = 2\cos x - 2\sin 2x$

$= 2\cos x(1 - 2\sin x)$

> 0 if $x \in (0, \pi/6) \cup (\pi/2, 5\pi/6) \cup (3\pi/2, 2\pi)$

< 0 if $x \in (\pi/6, \pi/2) \cup (5\pi/6, 3\pi/2)$